Math 115

Summer 2017

Lecture 16

$$\sqrt{xy}$$

① Simplify: $\dfrac{x+6}{x^2+12x+36} = \dfrac{1\;x+6}{(x+6)(x+6)} = \boxed{\dfrac{1}{x+6}}$

② Find all **excluded values**: $\dfrac{5}{x^2-3x-10}$

Deno. $=0$, Solve ↩

$x^2-3x-10=0$

$(x+2)(x-5)=0$

Z.F.P. $x+2=0$ $x-5=0$

$\boxed{x=-2}$ $\boxed{x=5}$ E.V. $-2,5$

① Simplify: $\dfrac{2x+6}{x^3+27} = \dfrac{2(\cancel{x+3})}{(\cancel{x+3})(x^2-3x+9)}$

$x^3 + 3^3$

$$= \boxed{\dfrac{2}{x^2-3x+9}}$$

② find all __excluded Values__: $\dfrac{3x-4}{25x^2-9}$

Deno. $= 0$, Solve

$25x^2 - 9 = 0$ ⟶ Use S.R.M.

$25x^2 = 9$ $\qquad x = \pm\sqrt{\dfrac{9}{25}}$

$x^2 = \dfrac{9}{25}$ $\qquad \boxed{x = \pm\dfrac{3}{5}}$ E.V.

$\pm\dfrac{3}{5}$

Simplify: $\dfrac{x^2-9}{x^2-4} \cdot \dfrac{x-2}{x+3} = \dfrac{(x+3)(x-3)}{(x+2)\cancel{(x-2)}} \cdot \dfrac{\cancel{x-2}}{\cancel{x+3}}$

$$= \boxed{\dfrac{x-3}{x+2}}$$

Simplify: $\dfrac{x^2-5x-24}{x^2-x-12} \div \dfrac{x^2-10x+16}{x^2+x-6}$

$= \dfrac{x^2-5x-24}{x^2-x-12} \cdot \dfrac{x^2+x-6}{x^2-10x+16}$

$= \dfrac{\cancel{(x-8)}\cancel{(x+3)}}{(x-4)\cancel{(x+3)}} \cdot \dfrac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}\cancel{(x-8)}} = \boxed{\dfrac{x+3}{x-4}}$

Simplify

$$\frac{x^2 + x - 42}{x^2 - 9} \div \frac{x^2 - 49}{x^2 - 6x + 9}$$

$$= \frac{(x+7)(x-6)}{(x+3)(x-3)} \cdot \frac{(x-3)(x-3)}{(x+7)(x-7)} = \boxed{\frac{(x-6)(x-3)}{(x+3)(x-7)}}$$

Adding & Subtracting like rational Exp. :

Same Deno.

$$\frac{x+4}{x^2-9} - \frac{1}{x^2-9} = \frac{x+4-1}{x^2-9}$$

$$\frac{2x-3}{x^2+x-6} - \frac{x-5}{x^2+x-6} \qquad = \frac{x+3}{x^2-9} = \frac{1(x+3)}{(x+3)(x-3)}$$

$$= \boxed{\frac{1}{x-3}}$$

$$= \frac{2x-3-x+5}{x^2+x-6} = \boxed{\frac{x+2}{x^2+x-6}}$$

$$= \boxed{\frac{x+2}{(x+3)(x-2)}}$$

Simplify

$$\frac{x+5}{x^2+14x+49} + \frac{2}{x^2+14x+49} = \frac{x+5 \ +2}{x^2+14x+49}$$

$$= \frac{x+7}{(x+7)(x+7)} = \boxed{\frac{1}{x+7}}$$

$$\frac{x^2}{x^2-9x+14} - \frac{7x}{x^2-9x+14}$$

$$= \frac{x^2-7x}{x^2-9x+14} = \frac{x(x-7)}{(x-7)(x-2)} = \boxed{\frac{x}{x-2}}$$

Simplify: $\dfrac{3x+8}{x^2+2x-15} - \dfrac{2x+3}{x^2+2x-15}$

$$= \frac{3x+8-(2x+3)}{x^2+2x-15}$$

$$= \frac{3x+8-2x-3}{x^2+2x-15} = \frac{x+5}{(x+5)(x-3)}$$

$$= \boxed{\frac{1}{x-3}}$$

$$\frac{5}{x^2+3x+2} + \frac{2}{x^2-4}$$

$$= \frac{5(x-2)}{(x+2)(x+1)(x-2)} + \frac{2(x+1)}{(x+2)(x-2)(x+1)}$$

$$= \frac{5(x-2)+2(x+1)}{(x+2)(x+1)(x-2)} = \boxed{\frac{7x-8}{(x+2)(x+1)(x-2)}}$$

Simplify

$$\frac{3}{x^2+x-6} - \frac{2}{x^2-9}$$

$$= \frac{3(x-3)}{(x+3)(x-2)(x-3)} - \frac{2(x-2)}{(x+3)(x-3)(x-2)}$$

$$= \frac{3(x-3)-2(x-2)}{(x+3)(x-2)(x-3)} = \boxed{\frac{x-5}{(x+3)(x-2)(x-3)}}$$

Simplify

$$\frac{x}{x^2+8x+15} - \frac{2}{x^2-x-12}$$

$$= \frac{x\,(x-4)}{(x+5)(x+3)(x-4)} - \frac{2\,(x+5)}{(x-4)(x+3)(x+5)}$$

$$= \frac{x(x-4)-2(x+5)}{(x+5)(x+3)(x-4)} = \boxed{\frac{x^2-6x-10}{(x+5)(x+3)(x-4)}}$$

Simplify

$$\frac{3}{x^2+2x-8} + \frac{2}{x^2-3x+2} - \frac{1}{x^2+3x-4}$$

$$= \frac{3(x-1)}{(x+4)(x-2)(x-1)} + \frac{2(x+4)}{(x-2)(x-1)(x+4)} - \frac{1(x-2)}{(x+4)(x-1)(x-2)}$$

$$= \frac{3(x-1)+2(x+4)-1(x-2)}{(x+4)(x-2)(x-1)} = \boxed{\frac{4x+7}{(x+4)(x-2)(x-1)}}$$

Simplify:

$$\frac{x+2}{3x^2+x-4} - \frac{x-3}{2x^2+x-3}$$

$$= \frac{(x+2)(2x+3)}{(3x+4)(x-1)(2x+3)} - \frac{(x-3)(3x+4)}{(2x+3)(x-1)(3x+4)}$$

$$= \frac{2x^2+3x+4x+6 - 3x^2-4x+9x+12}{(3x+4)(x-1)(2x+3)}$$

$$= \boxed{\frac{-x^2+12x+18}{(3x+4)(x-1)(2x+3)}}$$

Simplify

$$\frac{2x-5}{6x+9} - \frac{4}{2x^2+3x}$$

$$= \frac{(2x-5)\cdot x}{3(2x+3)\cdot x} - \frac{4\cdot 3}{x(2x+3)\cdot 3}$$

$$= \frac{2x^2-5x-12}{3x(2x+3)} = \frac{(2x+3)(x-4)}{3x(2x+3)}$$

$$= \boxed{\frac{x-4}{3x}}$$

Solving Rational Equations:

① Find the LCD and all excluded Values

② Use LCD to clear all denominators

③ Solve

④ Make sure not to include excluded Values if they appear in your Solns.

$x - \dfrac{12}{x} = 1$ $LCD = x$
$E.V.: 0$

$x \cdot x - x \cdot \dfrac{12}{x} = x \cdot 1$ → $(x-4)(x+3) = 0$

$x^2 - 12 = x$ $Z.F.P.$

$x^2 - 12 - x = 0$ $x-4=0$ $x+3=0$

$x^2 - x - 12 = 0$ $\boxed{x=4}$ $\boxed{x=-3}$

$\{4, -3\}$

Solve $\dfrac{2x}{x+4} + \dfrac{4}{x+4} = 3$ LCD = $x+4$

E.V.: -4

$(x+4) \cdot \dfrac{2x}{x+4} + (x+4) \cdot \dfrac{4}{x+4} = (x+4) \cdot 3$

$2x + 4 = 3(x+4)$

$2x + 4 = 3x + 12$

$2x - 3x = 12 - 4$

$-x = 8$

$\boxed{x = -8}$ $\{-8\}$

Solve :

1) $x - \dfrac{10}{x} = 3$

LCD = x
E.V.: 0

$x \cdot x - x \cdot \dfrac{10}{x} = x \cdot 3$

$x^2 - 10 = 3x$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$\boxed{x=5} \quad \boxed{x=-2} \quad \{5, -2\}$

2) $\dfrac{5x}{x+1} - \dfrac{3}{x+1} = 4$

LCD = $x+1$
E.V.: -1

$(x+1) \cdot \dfrac{5x}{x+1} - (x+1) \cdot \dfrac{3}{x+1} = (x+1) \cdot 4$

$5x - 3 = 4(x+1)$

$\boxed{x=7} \quad \{7\}$

Solve

$$\frac{x}{5} = \frac{5}{x}$$

LCD = 5x

E.V.: 0

$$\cancel{5x} \cdot \frac{x}{\cancel{5}} = 5\cancel{x} \cdot \frac{5}{\cancel{x}}$$

$$x^2 = 25 \quad \Big)\text{S.R.M.}$$

$$x = \pm\sqrt{25}$$

$$\boxed{x = \pm 5}$$

$$\{\pm 5\}$$

$$\frac{x-2}{7} = \frac{7}{x-2}$$

LCD = 7(x−2)

E.V.: 2

$$\cancel{7}(x-2) \cdot \frac{x-2}{\cancel{7}} = 7\cancel{(x-2)} \cdot \frac{7}{\cancel{x-2}}$$

$$(x-2)^2 = 49 \quad \Big)\text{S.R.M.}$$

$$x - 2 = \pm\sqrt{49}$$

$$x = 2 \pm 7$$

$$\boxed{x=9} \quad \boxed{x=-5} \quad \{9, -5\}$$

Solve $\dfrac{1}{x+2} + \dfrac{1}{x-2} = \dfrac{4}{x^2-4}$

LCD = (x+2)(x−2)

E.V.: −2, 2

$$\cancel{(x+2)}(x-2) \cdot \frac{1}{\cancel{x+2}} + (x+2)\cancel{(x-2)} \cdot \frac{1}{\cancel{x-2}} =$$

$$\cancel{(x+2)(x-2)} \cdot \frac{4}{\cancel{x^2-4}}$$

$$1(x-2) + 1(x+2) = 4$$

$$x \cancel{-2} + x \cancel{+2} = 4$$

$$2x = 4$$

$$\boxed{x = 2}$$

→ but 2 is an E.V.

$$\varnothing$$

Solve: $\dfrac{3}{x+3} + \dfrac{5}{x+4} = \dfrac{12x+19}{x^2+7x+12}$

$LCD = (x+3)(x+4)$

E.V.: $-3, -4$

$\cancel{(x+3)}(x+4) \cdot \dfrac{3}{\cancel{x+3}} + (x+3)\cancel{(x+4)} \cdot \dfrac{5}{\cancel{x+4}} = \cancel{(x+3)}\cancel{(x+4)} \cdot \dfrac{12x+19}{\cancel{x^2+7x+12}}$

$3(x+4) + 5(x+3) = 12x+19$

$3x+12+5x+15 = 12x+19$

$8x+27 = 12x+19$

$-4x = -8$

$\boxed{x=2}\ \checkmark$

$\{2\}$

Solve:

$\boxed{\dfrac{2}{x+2}} + \boxed{\dfrac{1}{x-2}} = \boxed{\dfrac{6}{x^2-4}}$ $\cancel{(x+2)}\cancel{(x-2)} \cdot \dfrac{6}{\cancel{x^2-4}}$

$LCD = (x+2)(x-2)$ E.V.: ± 2

$2(x-2) + 1(x+2) = 6$ $3x=8$

$2x-4+x+2 = 6$ $\boxed{x=\dfrac{8}{3}}$

$\left\{\dfrac{8}{3}\right\}$

The sum of [an integer] and [its reciprocal] is $\frac{5}{2}$. find the integer.

Let x be the integer,

$$x + \frac{1}{x} = \frac{5}{2} \qquad LCD = 2x , \quad E.V. \; 0$$

$$2x \cdot x + 2x \cdot \frac{1}{x} = 2x \cdot \frac{5}{2}$$

$$2x^2 + 2 = 5x$$

$$2x^2 - 5x + 2 = 0$$

$a = 2, \; b = -5, \; C = 2$

$$b^2 - 4ac = (-5)^2 - 4(2)(2)$$
$$= 25 - 16$$
$$= 9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{9}}{2(2)} = \frac{5 \pm 3}{4}$$

$$x = \frac{5+3}{4} = \frac{8}{4} = \boxed{2} \qquad x = \frac{5-3}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

$\{2\}$

$\frac{1}{2}$ is not an integer

The difference of reciprocals of two consecutive odd integers is $\frac{2}{15}$. Find all such integers.

Cons. odd integers : x , $x+2$

Reciprocals: $\frac{1}{x}$, $\frac{1}{x+2}$

$$\frac{1}{x} - \frac{1}{x+2} = \frac{2}{15}$$

LCD $= x(x+2)\cdot 15$, E.V. $0, -2$

$$15x(x+2)\cdot \frac{1}{x} - 15x(x+2)\cdot \frac{1}{x+2} = 15x(x+2)\cdot \frac{2}{15}$$

$$15(x+2) - 15x = 2x(x+2)$$

$$15x + 30 - 15x = 2x^2 + 4x$$

$$2x^2 + 4x - 30 = 0$$

Divide by 2 to reduce

$$x^2 + 2x - 15 = 0$$

$$x^2 + 2x - 15 = 0$$

$$x^2 + 2x + 1 = 15 + 1$$

$$(x + 1)^2 = 16$$

by S.R.M.

$$x + 1 = \pm\sqrt{16}$$

$$x + 1 = \pm 4$$

$$x = -1 \pm 4$$

$x = 3$, $x = -5$

x	$x+2$
3	5
-5	-3

The Sum of reciprocal of two Cons. even integers is $\frac{5}{12}$. find all Such integers.

$x, x+2$, $\frac{1}{x}$, $\frac{1}{x+2}$

$$\frac{1}{x} + \frac{1}{x+2} = \frac{5}{12}$$

LCD = $x(x+2) \cdot 12$

E.V. $0, -2$

$$12(x+2) + 12x = 5x(x+2)$$

$$12x + 24 + 12x = 5x^2 + 10x$$

$$5x^2 + 10x - 24x - 24 = 0 \qquad 5x^2 - 14x - 24 = 0$$

$a = 5 \qquad b = -14 \qquad c = -24$

$$b^2 - 4ac = (-14)^2 - 4(5)(-24)$$
$$= 676$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-14) \pm \sqrt{676}}{2(5)} = \frac{14 \pm 26}{10}$$

$$x = \frac{14+26}{10}$$

$$= \frac{40}{10} = 4$$

$$x = \frac{14-26}{10}$$

$$= \frac{-12}{10}$$

4 & 6

check

$$\frac{1}{4} + \frac{1}{6}$$

$$= \frac{10}{24} = \frac{5}{12}$$ ✓

what is LCM?

Least Common Multiple.

How to find LCM:
- Factor everything completely.
- Include all factors in your ans.

LCM: $2x+8$, $3x+12$

$$2x+8 = 2(x+4)$$
$$3x+12 = 3(x+4)$$

$$LCM = 2 \cdot (x+4) \cdot 3$$
$$\boxed{LCM = 6(x+4)}$$

find LCM : $2x-10$, $4x+20$, x^2-25

$2x-10 = 2 \cdot (x-5)$

$4x+20 = 2 \cdot 2 \qquad (x+5)$

$\dfrac{x^2-25 = \qquad (x-5)(x+5)}{\text{LCM } = 2 \cdot 2 \cdot (x-5)(x+5)}$
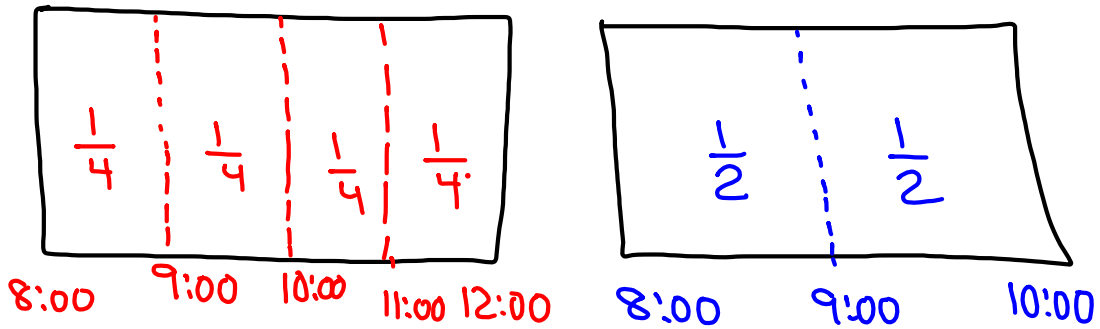
$= 4(x-5)(x+5)$

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find the LCM:

$3x^2-2x-5$, $2x^2+5x+3$

$3x^2-2x-5 = (3x-5)(x+1)$

$2x^2+5x+3 = \qquad (x+1)(2x+3)$

$\boxed{\text{LCM } = (3x-5)(x+1)(2x+3)}$

$\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$

8:00 9:00 10:00 11:00 12:00

$\frac{1}{2}$ | $\frac{1}{2}$

8:00 9:00 10:00

4 hrs

My rate $\frac{1}{4}$/hr

2 hrs

Your rate $\frac{1}{2}$/hr.

How long if we do this together?

Work Problems

My work + Your work = 1 Complete Work

How long?   How fast?

$$\frac{1}{4} \cdot t + \frac{1}{2} \cdot t = 1$$

$$\frac{t}{4} + \frac{t}{2} = 1$$

LCD = 4

$$4 \cdot \frac{t}{4} + 4 \cdot \frac{t}{2} = 4 \cdot 1$$

$t + 2t = 4$

$3t = 4$

$t = \frac{4}{3}$ hrs

$1\frac{1}{3}$ hrs

1 hr 20 Mins.

Pipe A can fill up an empty pool in 6 hrs.

Pipe B - - - - - - - - - - in 8 hrs.

How long if they are both working?

$$\text{work by Pipe A} + \text{work by Pipe B} = \frac{1}{\text{comp. work}}$$

$$\frac{1}{6} \cdot t + \frac{1}{8} \cdot t = 1$$
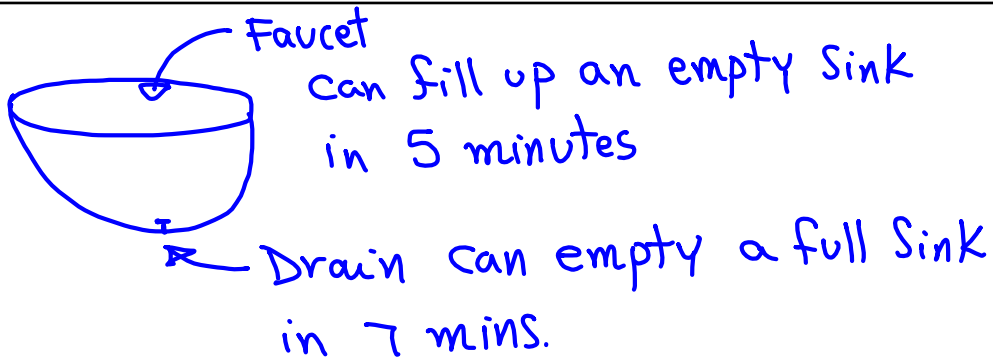
$$\frac{t}{6} + \frac{t}{8} = 1 \qquad LCD = 24$$

$$24 \cdot \frac{t}{6} + 24 \cdot \frac{t}{8} = 24 \cdot 1$$

$$4t + 3t = 24$$

$$7t = 24$$

$$t = \frac{24}{7} \qquad t \approx 3.4 \text{ hrs}$$

Faucet can fill up an empty Sink in 5 minutes

Drain can empty a full Sink in 7 mins.

Drain is open, water running, Sink is empty. How long to fill up the Sink?

$$\text{Work by faucet} - \text{Work by drain} = \underset{\text{Complete work}}{1}$$

$$\boxed{\frac{1}{5} \cdot t} - \boxed{\frac{1}{7} \cdot t} = \boxed{1}$$

LCD = 35

$$7t - 5t = 35$$
$$2t = 35$$
$$t = \frac{35}{2} \qquad t = 17.5 \text{ minutes}$$

It takes John to do a certain job
3 hrs longer than Mary.
They can do this job together in 2 hrs.
How long if they work alone?

Mary $\rightarrow x$ Rate $\rightarrow \dfrac{1}{x}$

John $\rightarrow x+3$ Rate $\rightarrow \dfrac{1}{x+3}$

$$\begin{array}{c}\text{work}\\\text{by}\\\text{Mary}\end{array} + \begin{array}{c}\text{work}\\\text{by}\\\text{John}\end{array} = \begin{array}{c}1\\\text{Complete}\\\text{work}\end{array}$$

$$\underset{\text{Rate}}{\boxed{\dfrac{1}{x}}} \cdot \underset{\text{Time}}{\boxed{2}} + \underset{\text{Rate}}{\boxed{\dfrac{1}{x+3}}} \cdot \underset{\text{Time}}{\boxed{2}} = 1$$

$$\boxed{\dfrac{2}{x}} + \boxed{\dfrac{2}{x+3}} = \boxed{1}$$

$LCD = x(x+3)$

$2(x+3) + 2x = x(x+3)$

$2x + 6 + 2x = x^2 + 3x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\Downarrow \qquad \Downarrow$

$x = 3 \qquad \cancel{x = -2}$

Mary   3 hrs
John   6 hrs

SG 18
Due tomorrow.